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**NEW YORK UNIVERSITY**  
FACULTY OF ARTS AND SCIENCE  
**DEPARTMENT OF APPLIED SCIENCE**



SEMI-ANNUAL STATUS REPORT  
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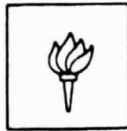
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## The Martian Climate: Energy Balance Models with CO<sub>2</sub>/H<sub>2</sub>O Atmospheres

### Progress Summary

The last semi-annual status report (March 1, 1984 - September 1, 1984) described the necessity to consider the meridional dependence of the Martian seasonal pressure wave. The basis is that atmospheric mass will be transported by the same eddy diffusive mechanisms as the transport of heat. This report presents a technique for the investigation, and develops coupled equations for mass and heat transport in a seasonal Mars model with condensation and sublimation of CO<sub>2</sub> at the polar caps. Up to the present, we have used the heat transport model to calculate latent-heat CO<sub>2</sub>-mass fluxes between ground and atmosphere with seasonal and latitude resolution. These fluxes will be taken into the mass transport model in the next phase of the program. The contents of this report are:

- (a) Physical Considerations of Planetary Mass and Energy Balance
- (b) Effects of Phase Changes at the Surface on Mass and Heat Flux
- (c) Atmospheric Transport and Governing Equations
- (d) Numerical Analysis

### Seasonal Mars Model: Formulation for a Volatile Planetary Surface with Atmospheric Transport of Mass and Heat Parameterized by Eddy Diffusion.

#### (a) Physical Considerations of Planetary Mass and Energy Balance

In formulating an energy balance climate model for Mars we characterize a vertical zonally-averaged column composed of both the carbon dioxide atmosphere and a thermally-interactive surface regolith layer by a single temperature  $T(t, \phi)$ , where  $\phi$  is the latitude and  $t$  is time. The atmosphere has an approximately exponentially decreasing density distribution  $\rho(t, \phi, z) \approx \rho_a(t, \phi, z)e^{-z/h}$  with altitude  $z$  from the surface at  $z = 0$  to  $z \rightarrow \infty$ , where  $\rho_a$  is the surface density and  $h$  the scale height; while the thermally-interactive surface extends from  $z = 0$  down to some thermal penetration depth  $z = d$ , where  $d = (2\kappa_s/\Omega)^{1/2}$ , where  $\kappa_s$  is the regolith (soil) thermal diffusivity and  $2\pi/\Omega$  is the period of the annual heating wave (Hoffert and Storch, 1979).

The atmospheric part exerts a surface pressure  $p(\phi, t)$  in general. The equation of state in the atmosphere is  $p = \rho_a RT$ , where  $R$  is the gas constant of CO<sub>2</sub>. Let:

$$m(\phi, t) \equiv \int_0^{\infty} \rho dz \approx \rho_a h \approx p/g \quad (1)$$

be the zonally-averaged mass of the atmospheric column, where the expression on the far RHS is from hydrostatic equilibrium, where  $g$  is the gravitational acceleration at Mars' surface. The corresponding atmospheric scale height is  $h \approx p/(\rho_a g) \approx RT/g$ . If  $c_a$  and  $c_s$  are the constant-pressure specific heats per unit mass of the atmosphere and regolith layers, the heat capacities per unit area of the atmosphere and "soil" per unit surface area area, respectively,  $mc_a = \rho_a c_a h$  and  $C_s = \rho_s c_s d$ , respectively.

### (b) Effects of Phase Changes at the Surface on Mass and Heat Flux

Let  $L \equiv RT^2 d \ln p / dT$  (Clausius-Clapeyron equation) be the latent heat of sublimation/condensation -- approximately constant for Mars conditions -- so the saturation vapor pressure over dry ice at the seasonal caps is

$$p(T) = (\text{constant}) \times e^{-L/RT}.$$

Over ice-free surfaces, the pressure is unconstrained by the underlying surface temperature, but is affected by transport from the seasonal caps which are continually subliming and condensing in response to seasonal insolation cycles. We assume  $\text{CO}_2$ -ice covered surfaces are in vapor pressure equilibrium with the atmosphere such that the ground temperature is

$$T_g = T_c \equiv -[(R/L) \ln(p/\text{constant})]^{-1} \approx 150 \text{ K} \quad (1)$$

whenever the surface is frost-covered. In particular, we set  $T_g = T$  when  $T > T_c$  and  $T_g = T_c$  when  $T < T_c$ . We also assume bare ground albedos  $\alpha_b$  and longwave emissivities  $\epsilon_b$  when  $T > T_c$  and different ice-covered values  $\alpha_i$  and  $\epsilon_i$  when  $T < T_c$ . If  $S(\phi, t)$  is the solar flux per unit area of the planet, and parameterizing the longwave cooling to space by a grey body approximation in terms of the ground temperature, we write the net radiative heating as

$$F^*(T, \phi, t) = \begin{cases} S(\phi, t)[1 - \alpha_b] - \epsilon_b \sigma T^4; & \text{if } T > T_c \\ S(\phi, t)[1 - \alpha_i] - \epsilon_i \sigma T_c^4; & \text{if } T < T_c \end{cases} \quad (2)$$

where  $\sigma$  the Stefan-Boltzmann constant. Since ground temperature is held

at  $T_c$  over ice, a sublimation mass flux  $m^*$  is injected locally into the atmosphere over ice to balance radiative and meridional eddy diffusive heating (regolith desorption and adsorption of carbon dioxide by processes other than sublimation/freezing are neglected). Assuming the sublimation heat flux  $Lm^*$  is supplied by the net radiative heating over ice  $F^*(T)$  plus a  $F_{diff}$  diffusive meridional heat flux (see equation 6) gives

$$m^*(T, \phi, t) = \begin{cases} 0; & \text{if } T > T_c \\ [S(\phi, t)[1 - \alpha_i] - \epsilon_i \sigma T_c^4 + F_{diff}]/L; & \text{if } T = T_c \end{cases} \quad (3)$$

where  $m^* = \partial m / \partial t = - \partial m_i / \partial t$ , in which  $m_i$  is the CO<sub>2</sub>-ice present on the ground. Notice that  $m^*$  can be either positive (sublimation) or negative (condensation), depending on whether solar heat or infrared cooling dominates  $F^*(T)$  over ice. The term  $F_{diff}$  will be zero at points over the ice cap, and will be positive only at the ice edge; also, it is small everywhere on Mars compared to the solar heat and infrared cooling. Here we couple the atmospheric temperature to the ice, so that  $T$  is never less than  $T_c$ .

### (c) Atmospheric Transport and Governing Equations

Meridional transport of both atmospheric mass and heat is by some wind field  $\mathbf{V} + \mathbf{V}'$ , where the "primed" component represents "eddy" fluctuations relative to the unprimed zonal-, altitude- and short period time-average velocity field. The eddies give rise to corresponding fluctuations in column mass and temperature which vary about the means, such that local values are  $m + m'$  and  $T + T'$ . By definition  $\langle \mathbf{V}' \rangle$ ,  $\langle m' \rangle$  and  $\langle T' \rangle$  are zero, but the eddy mass and heat fluxes  $\langle \mathbf{V}' m' \rangle$  and  $\langle \mathbf{V}' T' \rangle$  are not negligible, and may dominate the effects of the mean meridional flow for complex three-dimensional atmospheric circulations. Notice that both mass and heat fluxes are carried by the same velocity fields, and hence by the same eddy circulations, whose effects are subsumed under the  $\langle \rangle$ -average.

Integrating over the column and  $\langle \rangle$ -averaging the results then gives columnar conservation of mass and energy equations in the form:

$$\frac{\partial m}{\partial t} + \nabla \cdot [\mathbf{V}m + \langle \mathbf{V}' m' \rangle] = m^*(\phi, t)$$

$$[mc_a + C_s] \frac{\partial T}{\partial t} + mc_a \nabla \cdot [\mathbf{V}T + \langle \mathbf{V}'T' \rangle] + Lm^*(\phi, t) = F^*(\phi, t)$$

The divergence operator for a zonally-averaged planetary atmosphere treated as a thin spherical shell is:

$$\nabla \cdot (\ ) \equiv \frac{\partial [\cos \phi (\ )]}{a \cos \phi \partial \phi}$$

where  $a$  is the planetary radius.

It is likely that after vertical, horizontal and time averaging, the residual mean meridional velocity in the atmosphere is small compared to the eddy velocities,  $V \ll (V'^2)^{1/2}$ , as is the case with the earth's general circulation. This suggests a parameterization of the eddy heat fluxes in terms of an eddy diffusivity  $\kappa$  as in North et al (1980), James and North (1982), etc. However, and this is the critical new feature of this model, we assume that since atmospheric mass is transported by the same eddies as heat, that it has the same meridional eddy diffusivity, i.e.,

$$\kappa = \frac{-\langle V'm' \rangle}{\partial V / a \partial \phi} = \frac{-\langle V'T' \rangle}{\partial V / a \partial \phi}$$

Substituting these approximations, and defining a new meridional variable

$$x = \sin \phi, \quad (4)$$

proportional to the planetary surface area measured from the equator, gives (after some algebra), the conservation equations,

$$\frac{\partial m}{\partial t} = \frac{1}{\tau_m} \left[ (1-x^2) \frac{\partial^2 m}{\partial x^2} - 2x \frac{\partial m}{\partial x} \right] + m^*(T, x, t), \quad (5)$$

$$\frac{\partial T}{\partial t} = \frac{1}{\tau_T} \left[ (1-x^2) \frac{\partial^2 T}{\partial x^2} - 2x \frac{\partial T}{\partial x} \right] - \frac{Lm^*(T, x, t)}{C} + \frac{F^*(T, x, t)}{C}, \quad (6)$$

where  $C = mc_a + C_s$  is the heat capacity per unit surface area of the

atmosphere plus thermally-interactive regolith layer, and

$$\tau_m = a^2/\kappa; \quad \tau_T = (C/mc_a)a^2/\kappa \quad (7)$$

are timescales for meridional transport of atmospheric mass and heat. Notice  $\tau_T$  scales by an additional factor proportional to the ratio of atmospheric heat capacity to total heat capacity, which makes poleward heat transport relatively less important when the surface's heat capacity is large relative to the atmosphere's. With  $m^*(T,x,t)$  and  $F^*(T,x,t)$  given by equations (2) and (3) based on the local  $T(x,t)$ , the simultaneous solution of equations (6) and (7) by explicit finite-differences is straightforward.

Calculating  $m^*$  during condensation or sublimation of CO<sub>2</sub>-ice is accomplished as follows: If  $T = T_c$ , then first assume  $m^* = 0$ , and calculate  $\partial T/\partial t$  from equation (6). If this assumption results in  $\partial T/\partial t < 0$ , then condensation of CO<sub>2</sub>-ice is occurring. If, instead,  $\partial T/\partial t > 0$ , and, importantly,  $m_i > 0$ , then CO<sub>2</sub>-ice is subliming. In either case, the latent heat transfer holds the temperature constant. This procedure only determines that  $m^*$  is not zero, but does not calculate its value; this is accomplished by setting  $\partial T/\partial t = 0$ , and using equation (6) to calculate  $m^*$ . The only other possibility if  $T = T_c$  is that  $m_i = 0$ , and  $\partial T/\partial t > 0$  after assuming  $m^* = 0$  in equation (6); then, in fact,  $m^* = 0$  and equation (6) is used to calculate  $\partial T/\partial t$ .

(d) Numerical Analysis

Relevant parameter values for present Mars conditions as given by Hoffert et al. (1981):

$a$  = planetary radius  $\approx 3.39 \times 10^6$  m

$p_0$  = planetary mean surface pressure  $\approx 700$  Pa =  $700 \text{ kg/s}^2\text{-m}$

$g$  = surface gravitation acceleration  $\approx 3.73 \text{ m/s}^2$

$m_0$  = planetary mean atmospheric mass/unit area  $\approx 188 \text{ kg/m}^2$

$c_a$  = atmospheric heat capacity/unit mass  $\approx 830 \text{ J/kg-K}$

$c_a m_0$  = atmospheric heat capacity/unit area  $\approx 1.56 \times 10^5 \text{ J/m}^2\text{-K}$

The heat capacity per unit surface area of the ground estimated from the thermal inertia fit to temperature histories at the Mars lander sites is

$C_s$  = regolith heat capacity/unit area  $\approx 1.3 \times 10^6 \text{ J/m}^2\text{-K}$

The system heat capacity/unit area is therefore:

$C = C_s + c_a m_0 \approx 1.5 \times 10^6 \text{ J/m}^2\text{-K}$

And the atmosphere/surface heat capacity ratio is

$(c_a m_0 / C)_{\text{mars}} \approx 0.1$

This contrasts with the atmosphere/land heat capacity ratio of  $\sim 6$  on earth where the atmosphere is much thicker; but since the high heat capacity of the oceanic mixed layer dominates the net thermal inertia in the terrestrial case, the overall atmosphere/system heat capacity ratio on earth is (Hoffert et al., 1980):

$(c_a m_0 / C)_{\text{earth}} \approx 0.04,$

which is actually less than present-day mars.

Stone's (1974) atmospheric baroclinic instability model applied to mars conditions gives a meridional diffusion coefficient (Hoffert et al., 1981)  $D \equiv (m c_a / a^2) \kappa \approx 0.027 \text{ W/m}^2\text{-K}$ , and a corresponding meridional eddy diffusivity  $\kappa \approx a^2 D / m_0 c_a \sim 2 \times 10^6 \text{ m}^2/\text{s}$ . James and North (1982) suggest the lower value  $D \approx 0.004 \text{ W/m}^2\text{-K}$ , corresponding to  $\kappa \sim 3 \times 10^5 \text{ m}^2/\text{s}$ . For our present model we assume  $\kappa \sim 1 \times 10^6 \text{ m}^2/\text{s}$  for both heat and

mass transport, which is intermediate to these values.

The corresponding timescales in equations (5) and (6) are:

$$\tau_m = a^2/\kappa \approx 1.1 \times 10^7 \text{ s} \times \frac{1 \text{ mars year}}{5.9 \times 10^7 \text{ s}} \sim 0.2 \text{ mars years}$$

$$\tau_T = (C/mc_a) a^2/\kappa \approx 1.1 \times 10^8 \text{ s} \times \frac{1 \text{ mars year}}{5.9 \times 10^7 \text{ s}} \sim 2 \text{ mars years}$$

Notice that despite the same meridional diffusivities, meridional mass transport is some 10 times faster than meridional heat transport because the high thermal inertial of the regolith makes atmospheric transport relatively less important. Nevertheless, unlike James and North (1982) we do not assume this transport is infinitely fast ( $\tau_m \rightarrow \infty$ ), since it occurs over a significant fraction of the martian year. We show below that it also occurs over a time which is long compared with the timestep with which we must integrate the conservation equations, which indicates that equation (5) as well as equation (6) must be solved to find the seasonal pressure wave  $p(x,t) = m(x,t)/g$ .

Equations (5) and (6) can be written in terms of a generic variable  $q(x,t)$  as

$$\frac{\partial q}{\partial t} = \frac{1}{\tau_q} \left[ (1 - x^2) \frac{\partial^2 q}{\partial x^2} - 2x \frac{\partial q}{\partial x} \right] + q^*(T,x,t),$$

equal to  $m(x,t)$  or  $T(x,t)$ , where  $\tau_q$  is the relevant timescale for meridional mixing and  $q^*(T,x,t)$  is the relevant mass or heat source per unit area. In a finite-difference approximation, we assume the domain from  $x = -1$  to  $x = 1$  is discretized into  $K$  intervals of width  $\Delta x = 2/K$ , such that the  $J$ th point is located at  $x(J) = -1 + 2J/K$ , where  $J = 0, 1, 2, \dots, K$ , and  $J = 0, K$  are the endpoints at the south and north poles, respectively. Let  $I$  be an index denoting the time  $t = I\Delta t$ , where  $\Delta t$  is the timestep, from some initial time  $t = I = 0$  when the array  $q(0,J)$  is known. The object is to find  $q(I,J)$  at all future times. Assuming a time-explicit differencing scheme and centered differences then gives  $q(I+1,J)$  at all points except the endpoints  $I = 0$  and  $I = K$  in the form:

$$q(I+1,J) = q(I,J) + \frac{\Delta t}{\tau_q \Delta x^2} \cdot [A(J)q(I,J+1) + B(J)q(I,J) + C(J)q(I,J-1)] + \Delta t q^*(I,J),$$

where

$$A(J) = [1 - x(J)^2 - x(J)\Delta x],$$

$$B(J) = 2[x(J)^2 - 1],$$

$$C(J) = [1 - x(J)^2 + x(J)\Delta x].$$

The endpoints at  $J = 0$  and  $J = K$  are evaluated by recognizing that second derivatives drop out since they are multiplied by  $[1 - x(J)^2]$ , and by evaluating the first  $q$  derivative by one-sided differences:

$$q(I+1,0) = q(I,0) + [\Delta t/\tau_q][\{2[q(1) - q(0)]/\Delta x\}] + \Delta t q^*(I,0) \quad \text{for } x = -1$$

$$q(I+1,K) = q(I,K) - [\Delta t/\tau_q][\{2[q(K) - q(K-1)]/\Delta x\}] + \Delta t q^*(I,K) \quad \text{for } x = +1$$

The stability criteria for explicit solution of such parabolic PDEs is the Courant condition,  $\Delta t < 1/2\tau_q\Delta x^2$ . For a spatial stepsize of  $\Delta x = 0.1$  ( $K = 20$  points) we therefore require  $\Delta t < 0.0004$  mars years for equation (5) and  $\Delta t < 0.004$  mars years for equation (6). These timesteps are clearly much less than those required for mass (pressure) relaxation. Thus we need some 2500 computational steps/mars year to find the  $m(x,t)$  field but only 250 steps/mars year for the  $T(x,t)$  field. However, the mass field relaxes to repeating cycles over a time of 0.2 mars yr, while the temperature relaxes in about 2 mars years. This suggests a strategy in which the temperature field is computed first -- by running (say) 5 annual cycles to be conservative -- and the source term array  $m^*(I,J)$  from the last year is saved. The weak coupling of the  $m$ -equation to the  $T$ -equation through the dependence of  $\tau_T$  on the local  $m(x,t)$  may be neglected in view of uncertainties in  $\kappa$  by replacing  $m$  by the longterm mean value  $m_0$ . (In fact both  $\tau_m$  and  $\tau_T$  can probably be held constant in the calculation.) The mass field can then be computed with an order of magnitude smaller time resolution over (say) two annual cycles, saving only the last one.

Results can be plotted as contours of constant  $T$  and constant  $p = m/g$  on the  $t,x$  plane, or  $(L_S,x)$  plane where  $L_S$  is the aerocentric longitude, where intersections with the latitudes, or  $x$ 's, of the VL-1 and VL-2 landers give the time-histories of  $T(t)$  and  $p(t)$  at the lander sites. To show the seasonal forcing, it would be nice to see contours of constant insolation  $S$  on the  $(L_S,x)$  plane as well. Finally, the pressure histories from the lander barometers at the two Northern hemisphere locations should be

cross-plotted against observations and compared with the  $\tau_m \rightarrow 0$  approximation of James and North (1982).

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